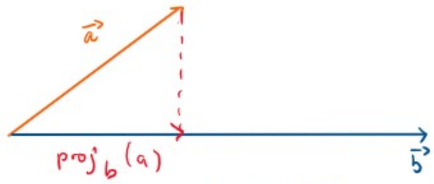
Phep chieu + Co so truc chuan

Phép chiếu a → b

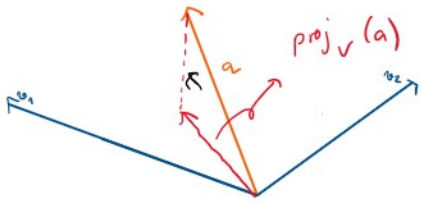
$$\mathrm{proj}_b(a) = \frac{a \cdot b}{\|b\|^2}\, b$$



$$a \cdot b = \|a\|\,\|b\| \cos(a, b)$$

$$\mathrm{proj}_b(a) = \|a\| \cdot \cos(a,b)\, \frac{b}{\|b\|} = \cancel{\|a\|}\, \frac{a \cdot b}{\cancel{\|a\|}\,\|b\|} \cdot \frac{b}{\|b\|} = \frac{a \cdot b}{\|b\|^2}\, b$$

Phép chiếu a → V có cơ sở $\{v_1, v_2, \dots, v_k\}$



$\mathrm{proj}_V(a)$

$$\mathrm{proj}_V(a) = a_1 v_1 + a_2 v_2 + \dots + a_k v_k$$

$$a - \mathrm{proj}_V(a) \perp v_i \quad (i = \overline{1,k})$$

$$\Leftrightarrow \left[a - (a_1 v_1 + a_2 v_2 + \dots + a_k v_k)\right] \cdot v_i = 0$$

$$\Leftrightarrow \sum_{m=1}^{k} a_m \underbrace{v_m \cdot v_i} = \underbrace{a \cdot v_i} \qquad \boxed{v_i \cdot v_j = 0}\ i \neq j$$

$$a_i \overbrace{(v_i \cdot v_i)}^{\|v_i\|^2} = a \cdot v_i$$

$$\Rightarrow a_i = \frac{a \cdot v_i}{\|v_i\|^2}$$

Nếu $\|v_i\|^2 = 1$

$$\to a_i = a \cdot v_i$$

Cơ sở trực giao, cơ sở trực chuẩn

Cơ sở $S \subset \mathbb{R}^k$ được gọi là trực giao nếu $v_i \cdot v_j = 0 \begin{cases} \forall i, j = \overline{1,k} \\ i \neq j \end{cases}$

Cơ sở $V \subset \mathbb{R}^k$ được gọi là trực chuẩn nếu V là cơ sở trực giao có $\|v_i\| = 1 \quad \forall i = \overline{1,k}$ → độc lập tuyến tính ?

Thuật toán Gram - Schmidt

• Cho cơ sở $\{b_1, b_2, \dots, b_k\}$ của $U \subset \mathbb{R}^n$

↳ Cơ sở trực chuẩn $V = \{v_1, v_2, \dots, v_k\}$?

(1) $v_1 := b_1$

$v_1 := \frac{v_1}{\|v_1\|}$ ←

(2) $v_2 := b_2 - \mathrm{proj}_{v_1}(b_2)$ $\qquad v_2 + \overset{v_1}{\mathrm{proj}_{v_1}(b_2)} = \overset{v_1, v_2}{b_2}$

$v_2 := \frac{v_2}{\|v_2\|}$ ←

② $v_2 := b_2 - \mathrm{proj}_{v_1}(b_2)$ $\qquad v_2 + \mathrm{proj}_{v_1}(b_2) = b_2$

$v_2 := \dfrac{v_2}{\|v_2\|}$ $\leftarrow$

. . .

ⓘ $v_i := b_i - \left[\mathrm{proj}_{v_1}(b_i) + \dots + \mathrm{proj}_{v_{i-1}}(b_i)\right]$

$v_i := \dfrac{v_i}{\|v_i\|}$ $\leftarrow$

↳ b_i tuyến tính $v_1, \dots v_i$

. . .

ⓚ $v_k := b_k - \left[\mathrm{proj}_{v_1}(b_k) + \dots + \mathrm{proj}_{v_{k-1}}(b_k)\right]$

$v_k := \dfrac{v_k}{\|v_k\|}$

V là hệ vector trực chuẩn $\mathrm{Span}\{V\} \supset \mathrm{Span}\{b_1, \dots, b_k\}$

V là cơ sở của không gian U ?

$\mathrm{proj}_{\mathrm{Span}\{v_1, \dots, v_{i-1}\}}(b_i)$

$v_i = b_i - \left[\mathrm{proj}_{v_1}(b_i) + \dots + \mathrm{proj}_{v_{i-1}}(b_i)\right]$

$\Leftrightarrow b_i = v_i + \mathrm{proj}_{\mathrm{Span}\{v_1, \dots, v_{i-1}\}}(b_i)$

$= v_i + c_1 v_1 + c_2 v_2 + \dots c_{i-1} v_{i-1}$

$\mathrm{Span}\{v_1, \dots v_i\}$

$\{b_1, \dots, b_k\}$ thuộc $\mathrm{Span}\{v_1, \dots, v_k\} = U$

↳ k

$\mathrm{Span}\{b_1, \dots, b_k\} = U$

Vd: Cho không gian $V = \{(x, y, z, t) \mid t = 0\}$ có cơ sở $S = \{a, b, c\}$

với $a = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix}$, $c = \begin{pmatrix} 3 \\ 4 \\ 5 \\ 0 \end{pmatrix}$

Cơ sở trực chuẩn $\{v_1, v_2, v_3\}$?

Vd: Cho vector $k = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$. Tìm hình chiếu của k lên

Vd1: $v_1 := a$ $\qquad \rightarrow v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} \vdots \\ 0 \\ 0 \end{pmatrix}$$

$$v_2 := b - \text{proj}_{v_1}(b)$$

$$\begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix} \to v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$v_3 := c - \left[\text{proj}_{v_1}(c) + \text{proj}_{v_2}(c)\right] \to v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 4 \\ 5 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 5 \\ 0 \end{pmatrix}$$

$$\{v_1, v_2, v_3\} \qquad \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

$\underline{\text{bc}}_2$:

$$k = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$\text{proj}_{\text{span}\{v_1, v_2, v_3\}}(k) = n_1 v_1 + n_2 v_2 + n_3 v_3$$

$$\Rightarrow \begin{cases} n_1 = k \cdot v_1 \\ n_2 = k \cdot v_2 \\ n_3 = k \cdot v_3 \end{cases} \Rightarrow \begin{cases} n_1 = x \\ n_2 = y \\ n_3 = z \end{cases}$$

$$\text{proj}_{\text{span}\{\}}(k) = \begin{pmatrix} x \\ y \\ z \\ 0 \end{pmatrix}$$