Ma trận

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

kích thước $m \times n$

kí hiệu $(a_{ij})_{m\times n}$

a_{ij} $\begin{cases} \text{dòng } i \\ \text{cột } j \end{cases}$

- Cộng ma trận

$$\underset{A}{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}} + \underset{B}{\begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}} = \underset{C}{\begin{bmatrix} (a_{11}+b_{11}) & (a_{12}+b_{12}) & \cdots & (a_{1n}+b_{1n}) \\ (a_{21}+b_{21}) & (a_{22}+b_{22}) & \cdots & (a_{2n}+b_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (a_{m1}+b_{m1}) & (a_{m2}+b_{m2}) & \cdots & (a_{mn}+b_{mn}) \end{bmatrix}}$$

$c_{ij} = a_{ij} + b_{ij}$ | A & B cùng chiều

- Nhân ma trận

αA

$$\alpha \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = \begin{bmatrix} \alpha a_{11} & \alpha a_{12} & \cdots & \alpha a_{1n} \\ \alpha a_{21} & \alpha a_{22} & \cdots & \alpha a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha a_{m1} & \alpha a_{m2} & \cdots & \alpha a_{mn} \end{bmatrix}$$

$\underset{m\times n}{A}\ \underset{n}{v} = K$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_n \\ | & | & & | \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

$$= v_1 \begin{pmatrix} | \\ a_1 \\ | \end{pmatrix} + v_2 \begin{pmatrix} | \\ a_2 \\ | \end{pmatrix} + \dots + v_n \begin{pmatrix} | \\ a_n \\ | \end{pmatrix}$$

$i \rightarrow$

$$\begin{bmatrix} k_1 \\ \vdots \\ k_m \end{bmatrix}$$

$$\begin{cases} \dim(K) = m \times 1 \\ k_1 = v_1 a_{11} + v_2 a_{12} + \dots + v_n a_{1n} \\ k_i = v_1 a_{i1} + v_2 a_{i2} + \dots + a_n a_{in} \end{cases}$$

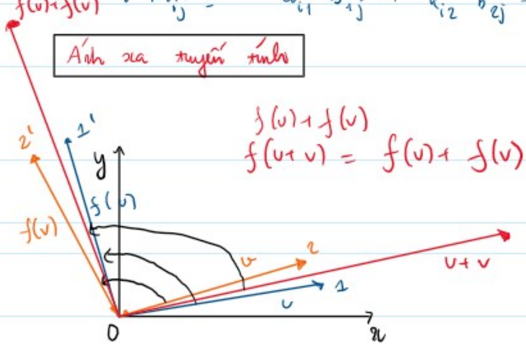
$\underset{m\times n}{A}\ \underset{n\times p}{B} = M$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{bmatrix}$$

$$\begin{bmatrix} m_{11} \\ \vdots \\ m_{m1} \end{bmatrix} \begin{bmatrix} m_{12} \\ \vdots \\ m_{m2} \end{bmatrix} \cdots \begin{bmatrix} m_{1p} \\ \vdots \\ m_{mp} \end{bmatrix}$$

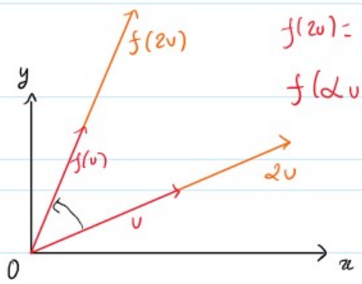
$$\begin{cases} \dim(M) = m \times p \\ m_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} + \dots + a_{in} b_{np} \end{cases}$$

Ánh xạ tuyến tính



$f(u) + f(v)$

$f(u+v) = f(u) + f(v)$



$f(2u) = 2f(u)$

$f(\alpha u) = \alpha f(u)$

Ánh xạ $f: \mathbb{R}^n \to \mathbb{R}^m$ là ánh xạ tuyến tính nếu

1. $\forall u_1, u_2 \in \mathbb{R}^n: f(u_1 + u_2) = f(u_1) + f(u_2)$
2. $\forall u \in \mathbb{R}^n, \alpha \in \mathbb{R}: f(\alpha u) = \alpha f(u)$

Cần biết bao nhiêu thông tin để xác định $f: \mathbb{R}^n \to \mathbb{R}^m$?

$\hookrightarrow f(e_i^n) \quad \forall i = \overline{1,n}$

$$e_i^n = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} \Longleftarrow i^{th}$$

$$f\left(\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}\right)$$

$f(u+v) = f(u) + f(v)$

$f(\alpha u) = \alpha f(u)$

$$m\left\{\overbrace{\begin{bmatrix} f(e_1) & f(e_2) & \dots & f(e_n) \end{bmatrix}}^{n}\right. \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = x_1 \begin{pmatrix} | \\ e_1 \\ | \end{pmatrix} + x_2 \begin{pmatrix} | \\ e_2 \\ | \end{pmatrix} + \dots + x_n \begin{pmatrix} | \\ e_n \\ | \end{pmatrix}$$

$$f\left(\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}\right) = f\left(x_1 \begin{pmatrix} | \\ e_1 \\ | \end{pmatrix} + x_2 \begin{pmatrix} | \\ e_2 \\ | \end{pmatrix} + \dots + x_n \begin{pmatrix} | \\ e_n \\ | \end{pmatrix}\right)$$

$$= f\left(x_1 \begin{pmatrix} | \\ e_1 \\ | \end{pmatrix}\right) + f\left(x_2 \begin{pmatrix} | \\ e_2 \\ | \end{pmatrix}\right) + \dots + f\left(x_n \begin{pmatrix} | \\ e_n \\ | \end{pmatrix}\right)$$

$$= x_1 f\begin{pmatrix} | \\ e_1 \\ | \end{pmatrix} + x_2 f\begin{pmatrix} | \\ e_2 \\ | \end{pmatrix} + \dots + x_n f\begin{pmatrix} | \\ e_n \\ | \end{pmatrix}$$

Ma trận biểu diễn

$$A(f) = \begin{bmatrix} | & | & & | \\ f(e_1^n) & f(e_2^n) & \dots & f(e_n^n) \\ | & | & & | \end{bmatrix} \in \mathbb{R}^{m \times n}$$

$$\mathbb{R}^p \xrightarrow{g} \mathbb{R}^n \xrightarrow{f} \mathbb{R}^m, \quad f \circ g = ?$$

$\left\{ \begin{array}{l} f \circ g \text{ ánh xạ tuyến tính} \\ A(f \circ g) = A(f)\, A(g) \end{array} \right.$

$$f \circ g \begin{pmatrix} x_1 \\ \vdots \\ x_p \end{pmatrix} = A(f \circ g) \begin{pmatrix} x_1 \\ \vdots \\ x_p \end{pmatrix} \quad (1)$$

$$\|$$

$$f\left(\underbrace{g\begin{pmatrix} x_1 \\ \vdots \\ x_p \end{pmatrix}}\right) = A(f) \cdot g\begin{pmatrix} x_1 \\ \vdots \\ x_p \end{pmatrix} = A(f)\, A(g) \begin{pmatrix} x_1 \\ \vdots \\ x_p \end{pmatrix} \quad (2)$$

$$A(f \circ g) = \underset{m \times n}{A(f)}\ \underset{n \times p}{A(g)}$$

Tính chất của phép nhân ma trận

$A(f) \cdot A(g) = A(f \circ g)$

$(AB)C = A(BC)$ $\qquad A, B, C \qquad f, g, h$

$(A+B)C = AC + BC$ $\qquad A(f \circ g)\, A(h) = A(f \circ g \circ h)$

$A(B+C) = AB + AC$ $\qquad A(f)\, A(g \circ h) = A(f \circ g \circ h)$

$(A+B)C = AC + BC \longrightarrow A(f\circ g)A(h) = A(f\circ g \circ h)$

$A(B+C) = AB + AC \qquad A(f)A(g\circ h) = A(f\circ g\circ h)$

$\boxed{\text{Phép toán trên ma trận}}$

Chuyển vị: $(a_{ij})^T_{mn} = (a_{ji})_{nm}$

$$m\left[\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}\right. \longrightarrow \left.\begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix}\right] n$$

$$\underbrace{\qquad\qquad}_{n} \qquad\qquad \underbrace{\qquad\qquad}_{m}$$

$v.t = v^T t = t^T v$

$(A+B)^T = A^T + B^T$

$(AB)^T = B^T A^T$

$\boxed{\text{Một số loại ma trận}}$

Ma trận đối xứng $\quad a_{ij} = a_{ji} \quad A^T = A$

Ma trận vuông $\quad A_{mn} \ \{ m = n$

Ma trận đường chéo $\quad \{ a_{ij} = 0 \ (i \neq j)$

Ma trận đơn vị $\quad \begin{bmatrix} 1 & & \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$

Ma trận tam giác

Ma trận không $\quad \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$

Nghịch đảo

Vd1: Tính phép nhân 2 ma trận $A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$ và $B = \begin{bmatrix} -\frac{5}{3} & \frac{4}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$

Cho $f: \mathbb{R}^n \to \mathbb{R}^n$. Nếu f song ánh $\Rightarrow f^{-1}$ tuyến tính $\quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Cho $A = A(f)$

$\Rightarrow A^{-1} = A(f^{-1})$

$AB = I$

A^{-1}

$AA^{-1} = I$

$(A^{-1})^{-1} = A$

$(AB)^{-1} = B^{-1}A^{-1}$

$(A^T)^{-1} = (A^{-1})^T$

$(A^{-1})^{-1} = A \qquad (AB)^{-1} = B^{-1}A^{-1}$ (nếu A & B khả nghịch)

$(A^T)^{-1} = (A^{-1})^T$

Vd2: Cho 2 ma trận $\begin{cases} A(m\times n) \\ B(p\times n) \end{cases}$ $(m \neq n, n \neq p, p \neq m)$

Phép toán nào sau đây có thể thực hiện?

$AB \quad A^TB \quad B^TA \quad BA \quad A^2$

$\times \quad \times \quad \times \quad \times \quad \times$

$AB^T \qquad BA^T$

Vd3: $\begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 2 & 1 & 2 \\ 3 & 4 & 5 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ -3 \\ 4 \end{bmatrix} \qquad \begin{bmatrix} 5 & 0 & -3 \\ 1 & 2 & 6 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 3 \\ 3 & 2 \\ 4 & 1 \end{bmatrix}$

$\begin{bmatrix} 3 & 4 & 5 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 4 \end{bmatrix}$ $\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$

Vd4: Cho không gian $\mathbb{R}^n$ có cơ sở trực chuẩn $\{v_1, \dots, v_n\}$

Nếu xếp $\underbrace{\begin{bmatrix} | & & | \\ v_1 & \dots & v_n \\ | & & | \end{bmatrix}}_{n} \Big\} n = A$

Chứng minh $A^T = A^{-1}$